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Unsteady Viscous Thin Airfoil Theory

by

J.E.Yates

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UNSTEADY VISCOUS THIN AIRFOIL THEORY,

by

(10) J.E. Yates

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PREFACE

During the last three years considerable progress has been made in the prediction of unsteady airloads on profiles in transonic flow, but prediction is still limited to potential inviscid flow. New analytical developments, and comparison with experimental results, need the introduction of the coupling between the inviscid flow and the boundary layer. Work in this field is at its beginning, but seems very promising and is being approached in many research laboratories. For these reasons, the Sub-Committee on Aeroelasticity of the Structures and Materials Panel considered it to be an appropriate time for the presentation of a pilot paper to assist in the preparation of a Specialists' Meeting in the Fall of 1980.

Dr Yates, in this publication, proposes an original approach to the problem which permits a first calculation of boundary layer effects. He derives a new kernel of the lifting surface integral equation which takes into account shear flow effects in the boundary layer. A very lively discussion took place after the presentation which was appreciated as an important step in the preparation of new Panel activities.

G.COUPRY
Chairman, Sub-Committee
on Aeroelasticity

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UNSTEADY VISCOUS THIN AIRFOIL THEORY

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SUMMARY

The concept of viscous thin airfoil theory introduced in Ref. 1 is formulated for unsteady incompressible flow. The theory is developed in detail for a flat plate airfoil with no thickness boundary layer. It is shown that the viscous pressure-downwash kernel function has a logarithmic singularity in contrast to the Cauchy singularity of inviscid theory. There is no eigensolution and no need for a Kutta condition to obtain a unique solution. It is shown by direct numerical solution that for Reynolds number greater than 1000 the viscous and inviscid results are virtually the same except in the immediate vicinity of the trailing edge. There, the pressure loading is greater than inviscid theory would indicate and the phase of the complex loading is less than inviscid theory. The results are in qualitative agreement with experimental results of Satyanarayana and Davis, Ref. 2. The effect of edge bluntness is illustrated for the case of steady flow.

LIST OF SYMBOLS

A_n	see Eqs. (5.1) and (5.2)
c	airfoil chord
C_{mn}	see Eqs. (5.2) and (5.3)
C_o , C	unperturbed and perturbed airfoil surface see Fig. 1
f	function used to define the normal displacement of the airfoil surface
F	normalized surface displacement
g_m	see Eq. (5.4)
$J_m(x)$	Bessel function
k	$\frac{\omega c}{2V_\infty}$ reduced frequency (based on semi-chord)
$K_o(z)$	modified Bessel function
K(x)	kernel function, see Eq. (4.19)
$K_1(x)$	integrated kernel function, see Eq. (4.20)
L(x)	lift distribution, see Eq. (4.8)
$\vec{n}$	unit vector normal to airfoil surface
P	dimensionless perturbation pressure
P*	conjugate pressure
P_o	pressure of unperturbed airfoil
V_∞	freestream velocity
Re	$\frac{V_\infty c}{\nu}$ Reynolds number
t	time
$\vec{t}$	unit vector tangent to airfoil surface
$T_n(x)$	Chebyshev polynomial of the first kind
$U_n(x)$	Chebyshev polynomial of the second kind
$\vec{v}_o$	potential flowfield of unperturbed airfoil

v_o^s unperturbed surface speed

$\vec{V}$ dimensionless perturbation velocity

x , y Cartesian coordinates, see Fig. 1

γ Eulers constant (0.57722)

ν kinematic viscosity

ρ_∞ freestream density

σ Re/4

σ^* $\sigma\left(1 + \frac{2ik}{\sigma}\right)^{\frac{1}{2}}$

ψ stream function

Special notation

c.p. center of pressure

C_L lift coefficient

C_{L_α} lift curve slope (per degree)

C_M moment coefficient (mid-chord reference)

div, grad, ∇^2 vector notation for divergence, gradient and Laplacian operators

$\frac{D_o}{Dt}$ $\frac{\partial}{\partial t} + \vec{v}_o \cdot$ grad

(') denotes perturbation quantity

Re , Im real and imaginary parts of a complex quantity

I. INTRODUCTION

The aeroelastic behavior of subsonic aircraft wings can be calculated with well established state of the art aerodynamic methods. Flutter speeds for practical aircraft wings are such that the reduced frequency is the order of a few tenths and the conventional unsteady aerodynamic analyses are adequate. We remark, however, that it is desirable to modify the steady state aerodynamic derivatives to agree with the "best available" steady loads data (Ref. 3). On the other hand, in hydroelastic applications, the reduced frequency can be of order unity and conventional unsteady aerodynamic methods are not always adequate. For example, flutter speed predictions for hydrofoils are at best unreliable (Refs. 4 and 5).

Several authors have attempted to explain some of the discrepancies between theory and experiment (Ref. 6). It is not our intent to give a comprehensive review of the aeroelastic work but to call attention to some of the suspected deficiencies in the aerodynamic theory. A significant improvement in flutter speed prediction was obtained by Carson Yates (Ref. 3) when steady loads data were used to correct the steady state aerodynamic derivatives. These results suggest a three-dimensional tip effect that is not accounted for in the standard linearized aerodynamic theory.

Other authors (Refs. 7 and 8) have focused on the Kutta condition, or rather its failure at higher reduced frequencies, as a prime suspect in the hydrofoil problem. Various suggestions have been made for emperically modifying the Kutta condition at higher reduced frequency. Also, several authors have inquired into the physical basis of the Kutta condition by including viscosity in the aerodynamic theory. This was accomplished by adopting the low Reynolds number incompressible model of Oseen to calculate the unsteady aerodynamic loads on a flat plate airfoil. The primary conclusion of Shen and Crimi (Ref. 8) was that the viscosity uniquely determines the circulatory part of the unsteady load. Furthermore, the loading is virtually the same as inviscid theory with Kutta condition with small corrections of order $(1/\sqrt{Re})$.

It is the purpose of this paper to re-examine and re-interpret the unsteady viscous problem in the light of more recent measurements of the unsteady pressure distributions on oscillating airfoils, and also to compare with some older steady state lift data. Some preliminary work on the steady state problem was reported in Ref. 1. Herein, we formulate the problem in terms of an integral equation (incompressible Possio) for the pressure. The viscous kernel is discussed in detail. Numerical results are presented for the flat plate airfoil at angle of attack, oscillating as a rigid body and subjected to a sinusoidal gust. Our results corroborate the general conclusions of previous authors who used the Oseen model but are re-interpreted for high Reynolds number turbulent flow.

II. CONCEPT OF VISCOUS THIN AIRFOIL THEORY

Consider a thin two-dimensional symmetric airfoil in an incompressible viscous flow at zero angle of attack Fig. 1a. We assume that the Reynolds number is very large ($> 10^6$) so that the boundary layer is turbulent and very thin. The pressure distribution can be calculated quite accurately with potential flow for the basic airfoil. Furthermore, if we add the boundary layer displacement thickness to the geometric thickness (dashed line in Fig. 1a) we can pose an equivalent potential flow problem. We solve for the potential flow on the mean boundary layer corrected airfoil shape. The surface of our corrected airfoil (denoted by C_o) actually moves at the potential flow velocity $\vec{t} \cdot \vec{v}_o^s$. The mathematical problem can be posed as follows:

$$\begin{aligned} \mathrm{div}\ \vec{v}_o &= 0 \\ \vec{v}_o \cdot \mathrm{grad}\ \vec{v}_o + \mathrm{grad}\ P_o &= \nu\nabla^2\vec{v}_o = 0 \\ \vec{v}_o &\sim \vec{v}_\infty \quad \text{at} \quad \infty \\ \vec{n} \cdot \vec{v}_o &= 0 \quad \text{on} \quad C_o \end{aligned} \tag{2.1}$$

The surface speed v_o^s is to be calculated from the solution of our equivalent potential flow problem.

Next, we suppose that the airfoil (now understood as the equivalent airfoil) surface is perturbed in some steady or unsteady way to shape C (Fig. 1b) that in general will produce lift. The perturbation flow (assumed to be small) satisfies the equations

$$\mathrm{div}\ \vec{v}' = 0$$

$$\frac{D_o\vec{v}'}{Dt} + \vec{v}' \cdot \mathrm{grad}\ \vec{v}_o + \mathrm{grad}\ P' = \nu\nabla^2\vec{v}' \tag{2.2}$$

and boundary conditions

$$\vec{v}_o + \vec{v}' = \vec{v}_o^s \quad \text{on} \quad C \tag{2.3}$$

$$\vec{v}' \sim 0 \quad \text{at} \quad \infty \tag{2.4}$$

The boundary condition, Eq. (2.3), can be transferred to the mean surface C_o to obtain

$$\vec{t}_o \cdot \vec{v}' = 0 \tag{2.5}$$

$$\vec{n}_o \cdot \vec{v}' = \frac{D_o f}{Dt} \quad \text{on} \quad C_o \tag{2.6}$$

where f is the local normal displacement of the surface.

The problem of "viscous thin airfoil theory" is to solve the viscous perturbation, Eqs. (2.2) subject to the boundary conditions, Eq. (2.5) and Eq. (2.6), and the infinity condition, Eq. (2.4). If we drop the viscous term in Eq. (2.2) and the perturbation no slip boundary condition, Eq. (2.5), the usual inviscid thin airfoil problem is recovered. However, some auxillary condition (e.g., Kutta) must be used to replace the missing viscous terms. Finally, we remark that an interesting and practical variation of the foregoing development is to replace the viscosity by the local eddy viscosity in Eq. (2.2).

III. THE FLAT PLATE AIRFOIL

For the remainder of this report we restrict our attention to the flat plate airfoil. Furthermore, we neglect the mean boundary layer displacement correction in which case

$$\vec{v}_o = \vec{v}_\infty = \vec{i}v_\infty \tag{3.1}$$

We choose the semi-chord $c/2$ as the unit of length, the freestream v_∞ as the unit of velocity and $\rho_\infty v_\infty^2$ as the unit of pressure. The dimensionless viscous thin airfoil problem for the flat plate becomes

$$\mathrm{div}\ \vec{V}' = 0$$

$$\frac{D\vec{V}}{Dt} + \mathrm{grad}\ P = \frac{1}{2\sigma}\nabla^2\vec{V} \tag{3.2}$$

$$\vec{i} \cdot \vec{V} = U(x,0\pm,t) = 0 \tag{3.3}$$

$$\vec{j} \cdot \vec{V} = V(x,0\pm,t) = \frac{DF}{Dt} \quad -1 < x < 1 \tag{3.4}$$

$$\vec{V} \sim 0 \quad \text{at} \quad \infty \tag{3.5}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \tag{3.6}$$

$$\sigma = Re/4 = \frac{v_\infty c}{4\nu} \tag{3.7}$$

and $F(x,t)$ is the deflection mode of the airfoil. Real quantities may be calculated with the relations,

$$\begin{aligned} \text{Time} &= \frac{c}{2v_\infty} \cdot t \\ \text{Distance} &= \frac{c}{2} \cdot \vec{x} \\ \text{Velocity} &= v_\infty \vec{V} \\ \text{Pressure} &= \rho_\infty v_\infty^2 P \end{aligned} \tag{3.8}$$

The momentum equation, Eq. (3.2), is formally identical to the Oseen equation. However, the justification for the use of the Eq. (3.2) for high Reynolds number is completely different from the low Reynolds number arguments that lead to the Oseen model. We have removed the symmetric boundary layer. Also, the perturbation viscous boundary condition, Eq. (3.3), is different from the full no-slip boundary condition used with the Oseen equation. The basic theory differs even further from the Oseen model if we use a Reynolds number, Eq. (3.7), based on a typical eddy viscosity. Many of the results we derive have a formal similarity to the results of Chu (Ref. 7) and Shen and Crimi (Ref. 8) who used Oseen's equation. However, the interpretation of our results is for high Reynolds number. Also, the theory can be extended in a logical self consistent way to account for geometric and boundary layer displacement thickness.

IV. DERIVATION OF AN EQUIVALENT INTEGRAL EQUATION

It is convenient to recast the viscous thin airfoil problem in the form of an integral equation. The procedure is much the same as with inviscid theory. For simplicity we consider simple harmonic motion and derive a fundamental solution corresponding to a pressure dipole located at the origin. By superposition of fundamental solutions we can write down the analog of Possio's integral equation for incompressible viscous flow.

For simple harmonic motion any dependent variable Q can be expressed in the form

$$Q = Re\left(Qe^{ikt}\right) \tag{4.1}$$

where

$$k = \frac{\omega c}{2v_\infty} \tag{4.2}$$

is the usual reduced frequency based on the semi-chord. We retain the same symbol Q for the complex amplitude of the dependent variables. The convective operator, Eq. (3.6), becomes

$$\frac{D}{Dt} = \frac{\partial}{\partial x} + ik \equiv \frac{D}{Dx} \tag{4.3}$$

Next, we introduce a streamfunction, ψ, such that

$$\vec{V} = \text{grad}\ \psi \times \vec{k} \tag{4.4}$$

satisfies the continuity equation. The momentum equation, Eq. (3.2), can be expressed in the simple form

$$\text{grad}\ P = \text{grad}\ P^* \times \vec{k} \tag{4.5}$$

where P^* is the conjugate pressure (Ref. 7)

$$P^* = -\frac{D\psi}{Dx} + \frac{1}{2\sigma}\nabla^2\psi \tag{4.6}$$

Both P and P^* are solutions of Laplace's equation and Eq. (4.5) is the vector form of the Cauchy Rieman equations. Thus,

$$P = P + iP^*$$

is an analytic function of the complex cariable $z = x + iy$. Below, we choose to work with the real functions P and P^*. However, it is worth noting that a complex representation for the pressure is possible even with viscous effects included. A complex analytic potential does not exist for the viscous problem.

By symmetry of the boundary conditions the pressure can be expressed as a superposition of dipoles along the airfoil chord; i.e.,

$$P = -\frac{1}{2\pi}\int_{-1}^{1} L(\xi)\frac{\partial}{\partial y}\ln R\, d\xi \tag{4.7}$$

where

$$L(x) = P(x,0^-) - P(x,0^+) \tag{4.8}$$

$$R = \sqrt{(x-\xi)^2 + y^2} \tag{4.9}$$

One can show by direct substitution into Eq. (4.5) that

$$P^* = -\frac{1}{2\pi}\int_{-1}^{1} L(\xi)\frac{\partial}{\partial x}\ln R\, d\xi \tag{4.10}$$

The streamfunction corresponding to our assumed pressure distribution is to be found by solution of Eq. (4.6). Consider the equation

$$\frac{1}{2\sigma}\nabla^2\psi - \frac{D\psi}{Dx} = \frac{\partial}{\partial x}\ln R \tag{4.11}$$

In the absence of viscosity the streamfunction is given by

$$\psi^{(0)} = -\int_{0}^{\infty} e^{-ik\xi}\frac{\partial}{\partial x}\ln R\, d\xi \tag{4.12}$$

so that

$$\nabla^2\psi^{(0)} = -2\pi\frac{\partial}{\partial x}\left[e^{-ikx}H(x)\delta(y)\right] \tag{4.13}$$

where $H(x)$ and $\delta(y)$ are the Heaviside step and Dirac delta functions respectively. The inviscid solution is a periodic vortex sheet that is shed from the dipole and extends unattenuated downstream to infinity. The effect of viscosity is to smooth out the discontinuity and cause the wake to decay.

Next, we write the solution of Eq. (4.11) in the form

$$\psi = \psi^{(0)} + \psi^{(1)} \tag{4.14}$$

where

$$\frac{1}{2\sigma}\nabla^2\psi^{(1)} - \frac{D\psi^{(1)}}{Dx} = \frac{\pi}{\sigma}\frac{\partial}{\partial x}\left[e^{-ikx}H(x)\delta(y)\right] \tag{4.15}$$

The solution can easily be expressed in a form similar to Eq. (4.12). The final form of ψ is

$$\psi = -\int_{0}^{\infty} e^{-ik\xi}\frac{\partial}{\partial x}\left[\ln R + e^{\sigma(x-\xi)}K_o(\sigma^* R)\right]d\xi \tag{4.16}$$

where

$$\sigma^* = \sigma\left(1 + \frac{2ik}{\sigma}\right)^{\frac{1}{2}} \tag{4.17}$$

and K_o is the modified Bessel function.

The perturbation velocity field corresponding to Eq. (4.15) is everywhere continuous. It follows by antisymmetry that the x-component of velocity is zero on the x-axis. Thus, the no-slip boundary condition, Eq. (3.3), is satisfied for an arbitrary line distribution of fundamental solutions. Also, the perturbation velocity decays at infinity so that Eq. (3.5) is satisfied. The final step is to superimpose the fundamental solutions and apply the boundary condition, Eq. (3.4), on the normal component of velocity. The final form of the integral equation for $L(x)$ becomes

$$\frac{1}{2\pi}\int_{-1}^{1} L(\xi)K(x-\xi)\,d\xi = -\frac{DF}{Dx} \tag{4.18}$$

with the kernel defined by

$$K(x) = \frac{\partial}{\partial x}K_1(x) \tag{4.19}$$

and

$$K_1(x) = \int_{0}^{\infty} e^{-ik\xi}\frac{\partial}{\partial x}\left[\ln|x-\xi| + e^{\sigma(x-\xi)}K_o(\sigma^*|x-\xi|)\right]d\xi \tag{4.20}$$

We refer to $K(x)$ as the kernel and $K_1(x)$ as the integrated kernel. The mathematical properties of $K(x)$ determine the basic character of the solution of Eq. (4.18) and are discussed below. For numerical purposes it is more convenient to evaluate the integrated kernel.

Discussion of the Kernel

The interpretation of the kernel is the same in viscous thin airfoil theory as it is in classical inviscid theory; i.e., it is the downwash velocity induced by a dipole pressure excitation at the origin. For high Reynolds number the most important effect of viscosity is near the origin. We can see this easily in the case of steady flow (Ref. 1) where

$$K(x) = \frac{1}{x} + \frac{\partial}{\partial x} e^{\sigma x} K_o(\sigma|x|) \sim - \sigma \ln \frac{\sigma|x|}{2}$$

$$+ O(\sigma^2|x| \ln \sigma|x|) \quad \text{for} \quad \sigma|x| \to 0 \tag{4.21}$$

The viscous kernel has a weak symmetric logarithmic singularity while the inviscid kernel has a Cauchy singularity. This statement is also true for the unsteady kernel. From the theory of integral equations of the first kind (see Ref. 9) it is known that Eq. (4.18) has a unique solution for a logarithmic kernel and has an eigensolution for a Cauchy kernel. In inviscid theory we fix the magnitude of the eigensolution by invoking the Kutta condition at the trailing edge. In viscous thin airfoil theory no auxiliary condition is required to solve the integral equation.

Another important property of the kernel is the difference between upstream and downstream influence. In steady flow, for example, the inviscid kernel is symmetric upstream and downstream. Viscosity causes a reduction in the downwash downstream of the load. Analytically the kernel is approximately

$$K(x) \sim \frac{1}{x}\left(1 - \frac{1}{2}\sqrt{\frac{\pi}{2\sigma x}}\right) \quad \sigma x \to +\infty$$

$$\sim \frac{1}{x} + e^{-2\sigma|x|}\sqrt{\frac{2\pi}{\sigma|x|}} \quad \sigma x \to -\infty \tag{4.22}$$

The steady state kernel is plotted in Fig. 2 as a function of σx. The nature of the singularity and the enhanced downstream influence are clearly evident in the viscous kernel.

The unsteady viscous kernel exhibits another fundamental difference from its inviscid counterpart. The inviscid vortex wake (downwash field) does not decay while the viscous wake decays exponentially like

$$e^{-k^2 x/2\sigma} \tag{4.23}$$

(see Fig. 3). This feature of the kernel is physically appealing in that a real wake does decay ultimately due to turbulence. If we base the Reynolds number on a typical wake eddy viscosity then the decay formula, Eq. (4.23) becomes even more meaningful. The effect of Reynolds number on the integrated unsteady kernel near the origin is illustrated in Fig. 4. For fixed $x > 0$ the difference between the viscous and inviscid kernel vanishes like $1/\sqrt{Re}$. Upstream, the convergence is exponential.

V. NUMERICAL RESULTS

To solve the basic integral equation (4.18), we expand the load distribution $L(x)$ in a Chebyshev series; i.e.,

$$L(x) = \sum_{n=0}^{N} A_n \frac{T_n(x)}{\sqrt{1 - x^2}} \tag{5.1}$$

where $T_n(x)$ is the Chebyshev polynomial of the first kind (Ref. 10). Substitute Eq. (5.1) into Eq. (4.18), multiply by $U_m(x)$ (Chebyshev polynomial of the second kind) and integrate form -1 to 1. The result is a matrix equation for the unknown coefficients, A_n; i.e.,

$$\sum_{n=0}^{N} A_n C_{mn} = g_m \qquad m = 0,1,2,..,N \tag{5.2}$$

where

$$C_{mn} = \frac{(m + 1)}{\pi^2} \int_{-1}^{1} \frac{T_{m+1}(x)}{\sqrt{1 - x^2}} dx \int_{-1}^{1} \frac{T_n(\xi)}{\sqrt{1 - \xi^2}} K_1(x - \xi)d\xi \qquad (5.3$$

and

$$g_m = -\frac{2}{\pi}\int_{-1}^{1}\sqrt{1-x^2}\ U_m(x)\ \frac{DF}{Dx}\ dx \tag{5.4}$$

Three special cases of the mode shape, downwash and the coefficients g_m are noted below:

Translation:

$$F(x) = 1 \quad , \quad \frac{DF}{Dx} = ik$$

$$g_o = -ik \ , \quad g_m = 0 \quad , \qquad m = 1,2,... \tag{5.5}$$

Pitch about Mid-Chord

$$F(x) = -x \quad , \quad \frac{DF}{Dx} = -(1 + ikx)$$

$$g_o = 1 \quad , \quad g_1 = \frac{ik}{2} \ , \ g_m = 0 \ , \quad m = 2,3,... \tag{5.6}$$

Sinusoidal Gust:

$$\frac{DF}{Dx} = -e^{-ikx}$$

$$g_m = 2(m + 1)(-i)^m \frac{J_{m+1}(k)}{k} \quad , \quad m = 0,1,... \tag{5.7}$$

where J_m is the Bessel function of the first kind. The coefficients C_{mn} are functions of the reduced frequency and the Reynolds number and must be evaluated numerically. We remark that it is convenient to introduce the trigonometric substitution in Eq. (5.3)

$$x = \cos\theta \quad , \quad \xi = \cos\phi \tag{5.8}$$

in which case

$$C_{mn} = \frac{(m + 1)}{\pi^2}\int_0^{\pi}\cos\ (m+1)\theta\ d\theta\int_0^{\pi}\cos\ n\phi\ d\phi\ K_1(\cos\theta - \cos\phi) \tag{5.9}$$

The integrated viscous kernel is regular at the origin so that the integrals in Eq. (5.9) can be evaluated by any standard quadrature routine. While the general theory of integral equations of the airfoil type is not our main concern in this paper, we point out that Williams (Ref. 11) has derived a general solution of Eq. (4.18) in terms of two fundamental solutions. The latter correspond essentially to the cases of translation and pitch about mid-chord. Also, Williams points out (Ref. 12) that the evaluation of matrix coefficients like Eq. (5.9) can be reduced to a single quadrature over the Fourier transform of the kernel function. This might be a more practical method than the direct evaluation of the kernel.

For any finite N and in the absence of viscosity, the matrix C_{mn} is singular and has the rank N-1 . There is one eigensolution whose amplitude is fixed by invoking the Kutta condition at the trailing edge. In practice the Kutta condition is satisfied by choosing a set of pressure mode shapes that individually vanish at the trailing edge. In Eq. (5.1) we have allowed for the possibility of a square root singularity in the pressure loading at the leading and trailing edges. With the viscous kernel the matrix C_{mn} is not singular and there is no eigensolution. The circulatory component of the unsteady loading is uniquely determined for any finite Reynolds number. We are of course rephrasing in matrix terms our earlier observation that the integral equation, Eq. (4.18), has a unique solution for the viscous kernel because of the logarithmic singularity.

It is a curious theoretical fact (Ref. 9) that the inclusion of viscosity in the analysis does not alter the square root edge singularity in the pressure. Only by including the nonlinear effect of edge bluntness can this singularity be removed.

The lift and moment coefficients and center of pressure are readily calculated in terms of the Chebyshev coefficients; i.e.,

$$C_L = \frac{L}{1/2\rho_\infty v_\infty^2 c} = \pi A_0$$

$$C_M = \frac{M}{1/2\rho_\infty v_\infty^2 c^2} = \frac{-\pi A_1}{4}$$

$$c.p. = -2C_M/C_L = \frac{1}{2}\frac{A_1}{A_0} \tag{5.10}$$

In Fig. 5, the flat plate lift curve slope calculated from Eq. (4.18) is plotted as a function of Reynolds number and compared with the limit case of inviscid theory plus the Kutta condition. The viscous flat plate solution converges to the inviscid limit as $1/\sqrt{Re}$. For $Re > 1000$, the viscous and inviscid results are virtually the same. These numerical results are in agreement with the conclusion of Shen and Crimi (Ref. 8). Even though we have allowed for the possibility of a square root singularity in the loading at the trailing edge, the effect of viscosity is to remove the singularity in accordance with the empirical Kutta condition.

The insight that the viscous analysis provides is that the "Kutta condition" is a consequence of the integrated effect of viscosity over the entire wetted surface of the airfoil. This point can be made stronger in the case of unsteady flow. For example, in Figs. 6a and 6b we compare our theory with the experimental results of Satyanarayana and Davis (Ref. 2). There is no difference between the viscous and inviscid theory except over roughly the last 5% of the airfoil. There the magnitude of the load is greater than the inviscid result that drops to zero as the square root of the distance from the trailing edge in accordance with the Kutta condition. Also, there is a drop in the phase of the loading. These results are in qualitative agreement with the experimental results. However, the experiments indicate much greater deviations from local Kutta behavior in particular at the reduced frequency of 1.23 (see Fig. 6b).

The main point is that the loading over the majority of the frontal portion of the airfoil can be calculated quite accurately with inviscid theory plus the Kutta condition even though the local behavior of the loading at the trailing edge may in no way resemble the potential flow. It is the integrated effect of viscosity that is responsible for the circulatory component of the steady or unsteady loading. This is the reason why local modifications (e.g., bluntness) to an airfoil trailing edge can be made without making gross changes in its overall lift performance (Ref. 13).

A final result to illustrate our main point is shown in Fig. 7 where we subject an airfoil to a sinusoidal gust at a reduced frequency of 2.0. Again, the viscous and inviscid loadings are indistinguishable over the front portion of the airfoil. Near the trailing edge the load increases and the phase dips below its inviscid counterpart. Recent measurements of unsteady pressure due to buffeting (Ref. 14) indicate that the enhanced trailing edge loading does occur.

Effect of Edge Bluntness

In Ref. 1, we presented some results for steady flow that illustrate the importance of leading edge bluntness. The measured lift curve slope of a large collection of standard airfoils (Ref. 15) is 15 to 20% less than the value predicted by inviscid theory with Kutta condition. Typical experimental results for Reynolds numbers between 10^6 and 10^7 are shown in Fig. 8. The theoretical result was calculated with an ad hoc modification of viscous thin airfoil theory that in essence suppresses the effect of viscosity in the immediate vicinity of the leading edge. The argument is that if the leading edge were blunt like an actual subsonic airfoil, then the square root leading edge singularity of the pressure would not be present. The effect of viscosity combined with a sharp leading edge is to grossly overestimate the leading edge load and total circulation. Our ad hoc correction suppresses this singular effect and yields a formula for the lift curve slope that has a logarithmic Reynolds number correction. The order of magnitude and trend of the correction is in very good agreement with experimental data as shown in Fig. 8. Because of the success of our ad hoc correction, considerable emphasis is being placed on a more rational correction for edge bluntless in our current research work.

Comments on Asymptotic Trailing Edge Theories

In recent years several papers (Refs. 16, 17, 18) have considered the detailed flow around an airfoil with a mathematically sharp trailing edge. The problem is to match in every detail an inner viscous boundary layer flow to an external potential flow. Multi-layered asymptotic analysis is necessary to complete the matching. The most practical application of this type of analysis is the recent strong interaction theory of Melnik (Ref. 18). The lift and drag of an airfoil in steady compressible flow with an attached turbulent boundary layer can now be calculated with great accuracy.

All of these analyses rely heavily on a highly detailed asymptotic analysis of the trailing edge region to connect the boundary layers on the upper and lower surfaces. The trailing edge must be mathematically sharp and in the innermost solution near the trailing edge the "least singular" branch of the possible solutions is chosen. This is equivalent to applying a "Kutta" condition. It seems ironical that an empirical mathematical statement must be made to establish the uniqueness of such a beautiful asymtotic theory. Yet, we can illustrate very simply why such a condition must be used with conventional boundary layer analysis. If we replace the viscous term ($\nu\nabla^2\vec{v}$) in our Eq.(2.2) with its "parabolized" boundary layer counterpart $\left(\nu \frac{\partial^2 \vec{v}'}{\partial y^2}\right)$ then it is not difficult to show (we omit the proof here) that the resulting kernel function in our viscous thin airfoil theory still has a Cauchy singularity. The solution for the lift distribution will have an eigensolution and some auxiliary criteria (Kutta) must be used to establish a unique solution. The full elliptic behavior of the viscous term must be retained in the Navier Stokes equations if the need for a Kutta condition is to be eliminated. There is no reason to suspect that the nonlinear terms in the fluid equations can change this requirement.

Finally, we remark that the concept and spirit of viscous thin airfoil theory is completely different from the asymptotic theories. Our basic working principle is that the most important viscous corrections in the gross aerodynamic properties needed for aeroelastic calculations can be calculated without recourse to highly detailed boundary layer analysis. To the extent that the Kutta condition is in itself the most important viscous correction, the principle has already served us well.

VI. CONCLUSIONS

We have formulated the concept of unsteady viscous thin airfoil theory and applied the theory to a flat plate airfoil. For this special case our results are formally equivalent to those of Chu (Ref. 7), and Shen and Crimi (Ref. 8), who use the Oseen equation. The present theory is valid for high Reynolds number turbulent flow in contrast to the Oseen model.

The main conclusion of our work is that viscosity is the most important missing element of physics in our potential flow theories. Without viscosity one must invoke an auxillary condition such as the Kutta condition to determine the circulatory component of the loading. The viscous theory indicates that the development of circulation is due to the integrated effect of viscosity over the entire wetted surface of the airfoil. Local deviations from smooth Kutta flow at the trailing edge are quite likely to occur on any real airfoil without altering the overall aerodynamic loads required in an aeroelastic analysis.

An ad hoc correction for leading edge bluntness is presented for the case of steady flow. The correction to the lift curve slope is of order $1/\ell n$ Re and agrees in magnitude and trend with high Reynolds number steady state lift curve data. We conclude that leading edge bluntness must be accounted for in viscous thin airfoil theory to avoid a severe overestimation of the leading edge load and total circulation.

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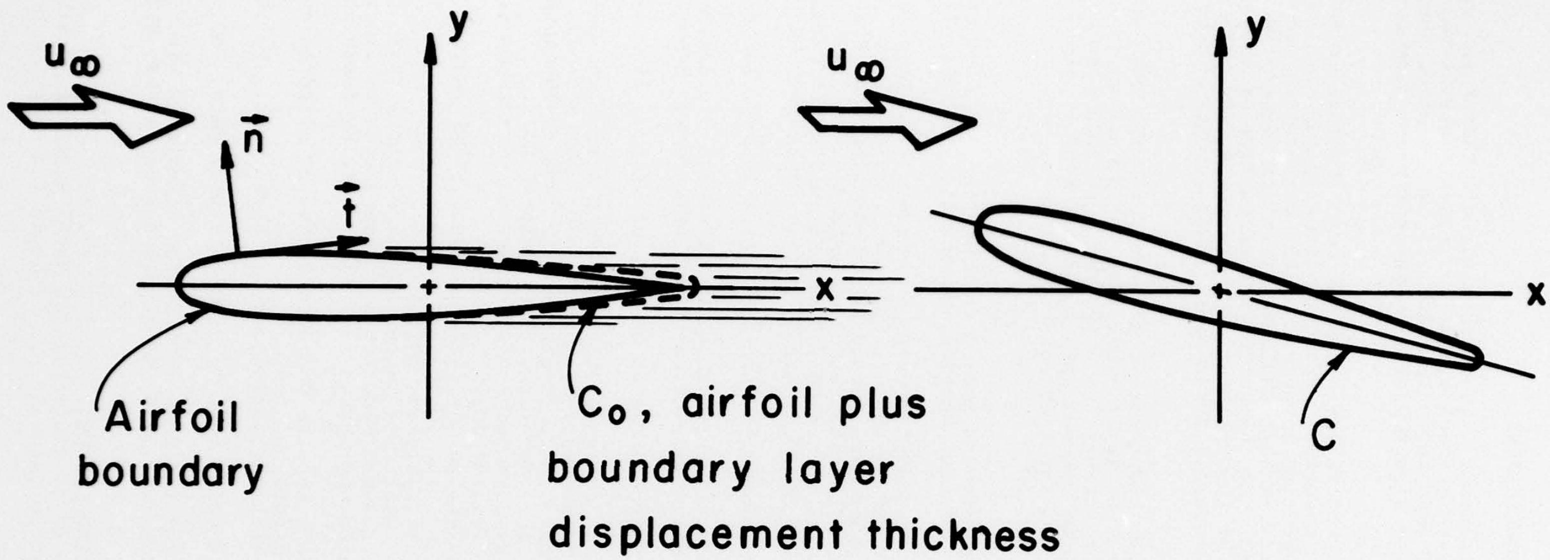

(a) Airfoil at zero angle of attack with mean boundary layer

(b) Perturbed equivalent airfoil shape

Figure 1. Geometry of the viscous thin airfoil problem and removal of the symmetric boundary layer.

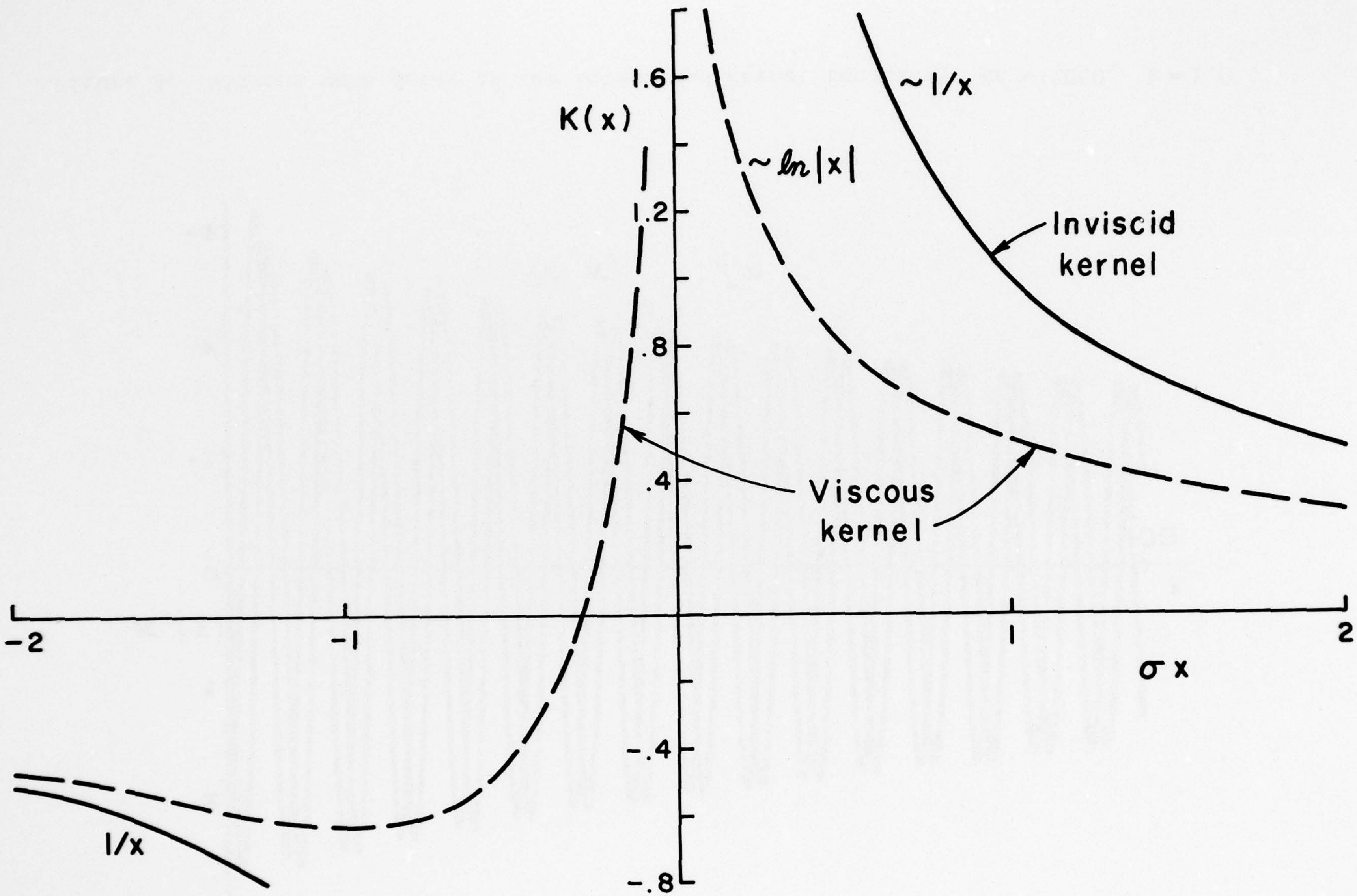

Figure 2. Viscous kernel function of the steady incompressible airfoil equation

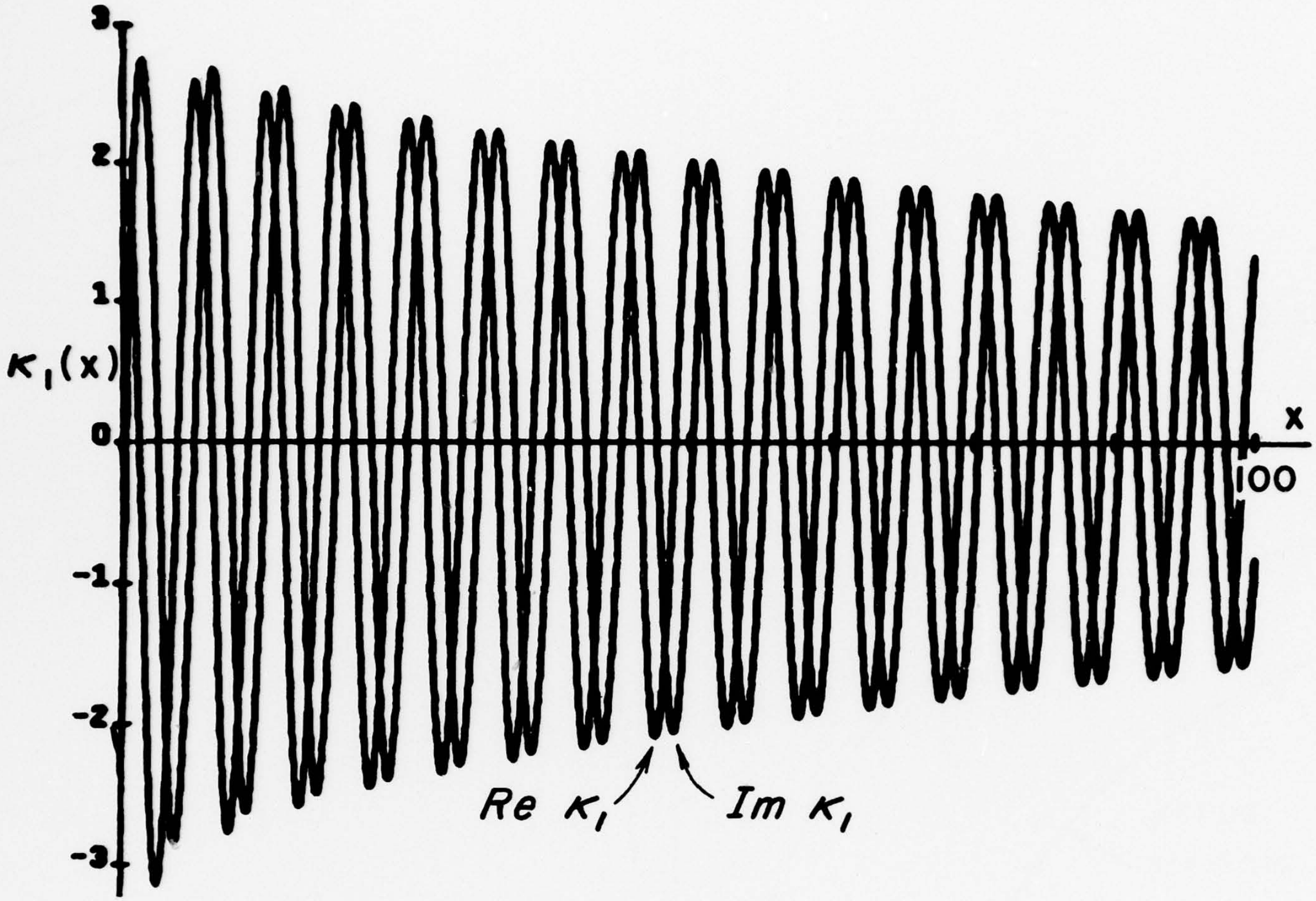

Figure 3. Viscous wake decay of the integrated kernel function; Re = 1000, k = 1.0 .

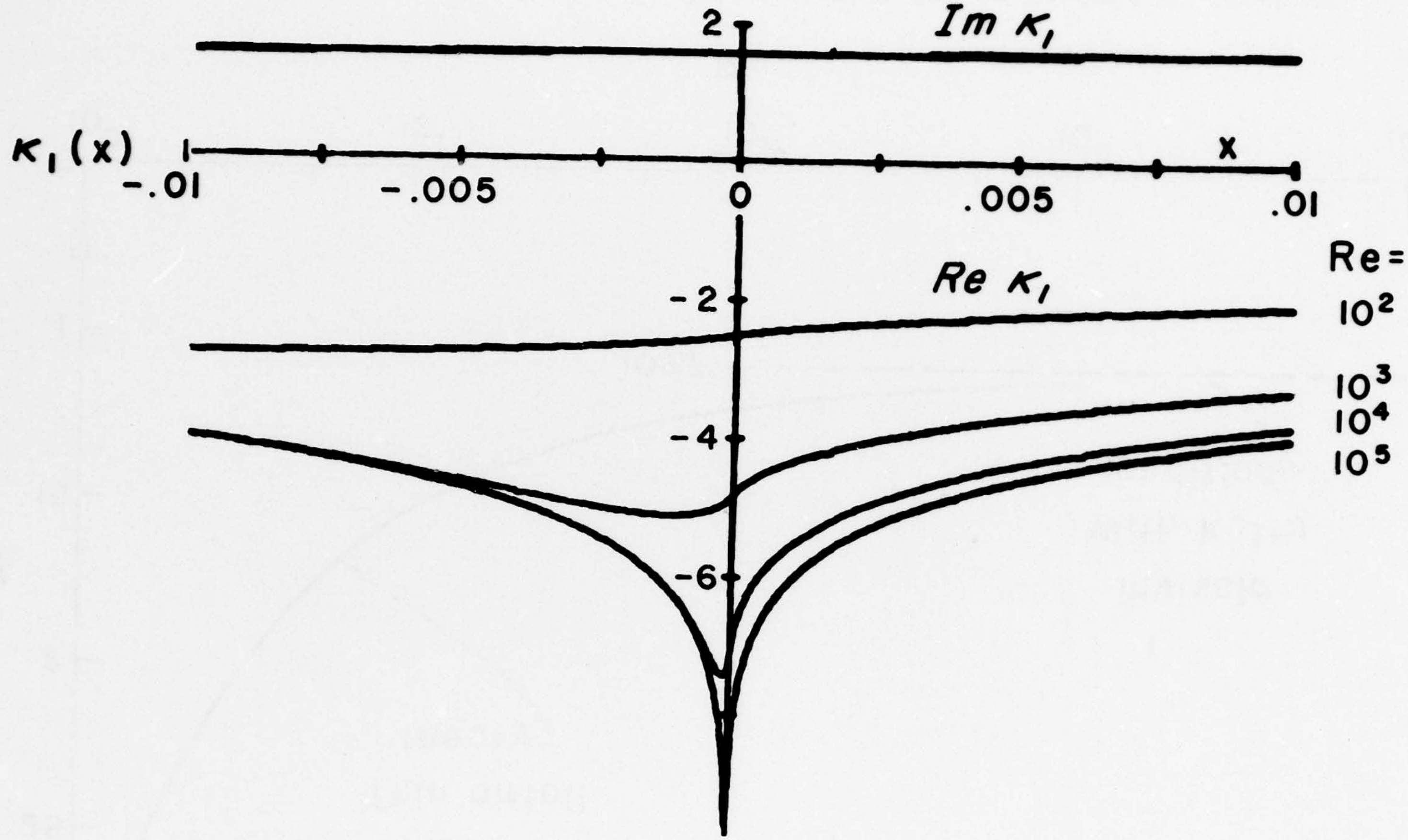

Figure 4. Effect of Reynolds number on the integrated kernel near the origin; $k = 1.0$.

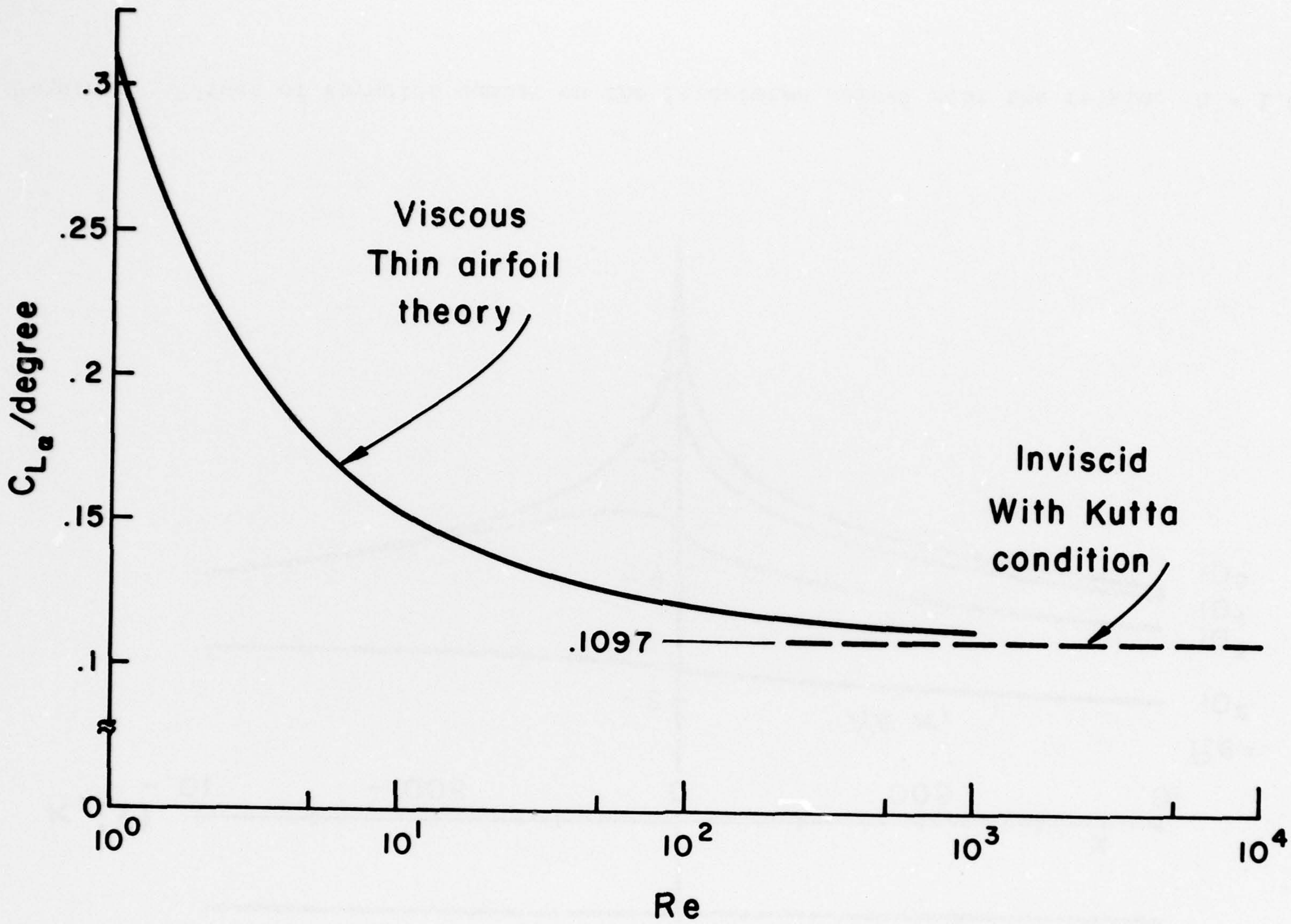

Figure 5. Solution of the viscous airfoil equation for an airfoil at constant angle of attack.

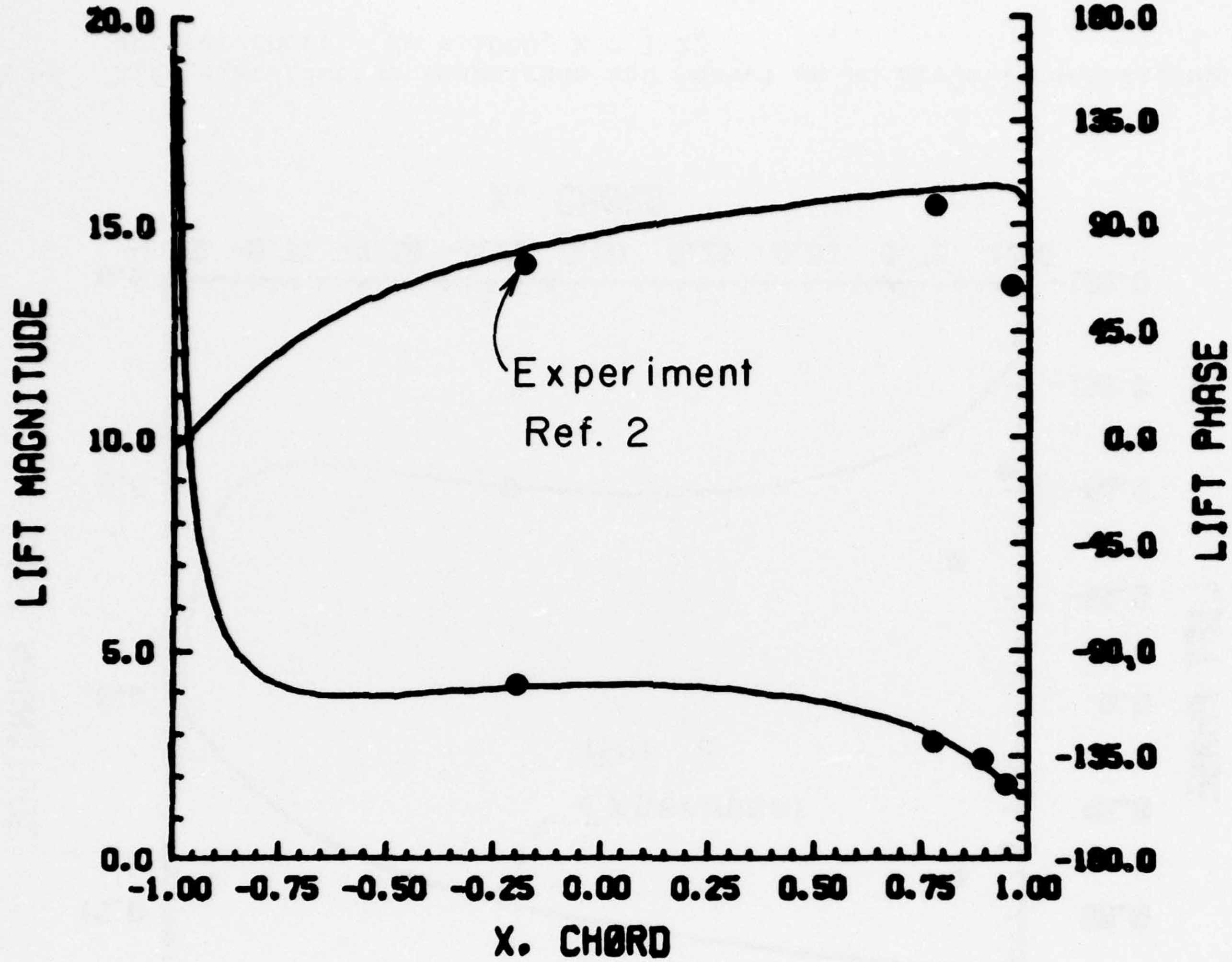

Figure 6a. Lift distribution (magnitude and phase) on an airfoil oscillating about quarter chord; Re = 1000, k - 1.02 .

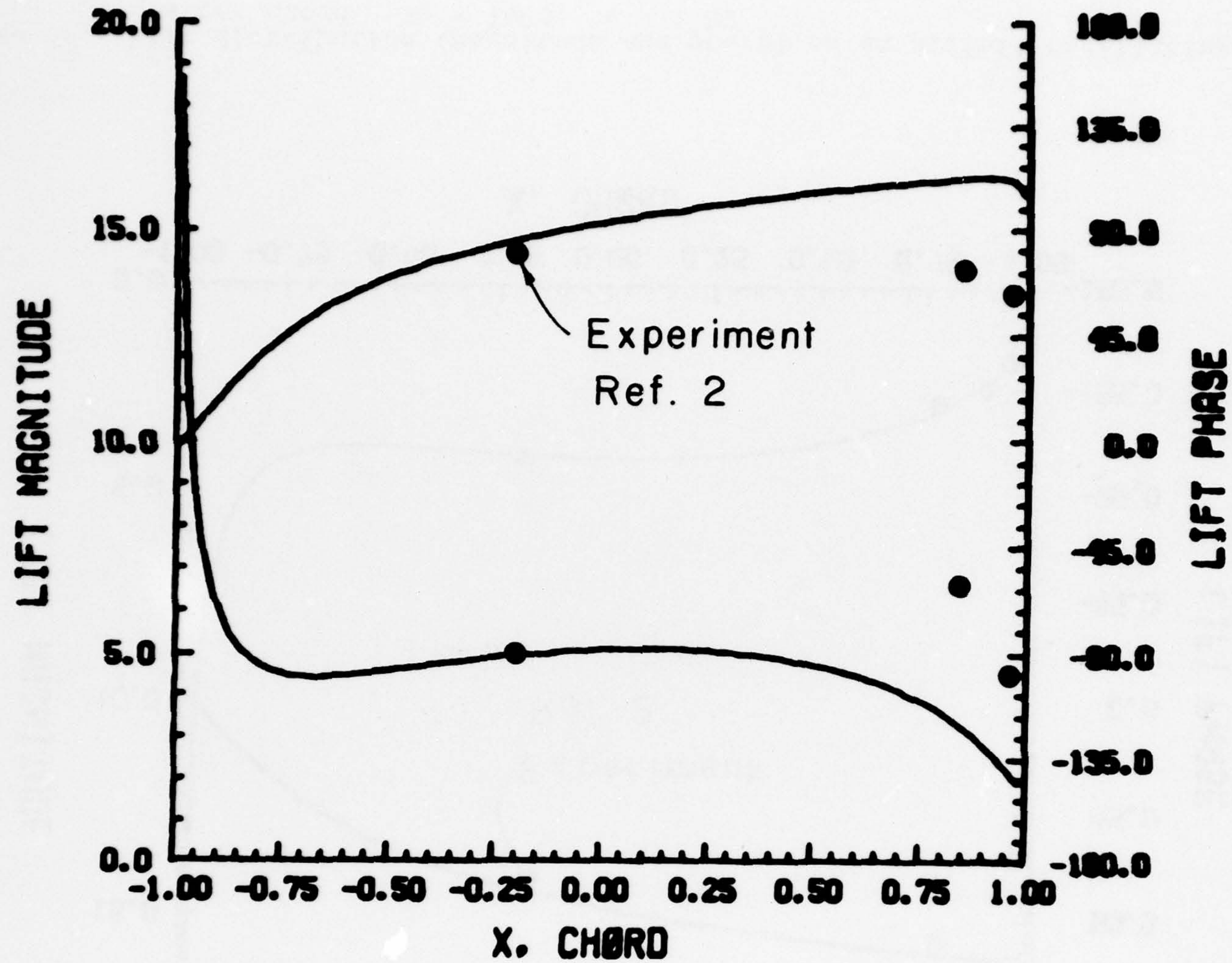

Figure 6b. Lift distribution (magnitude and phase) on an airfoil oscillating about quarter chord; Re = 1000, k = 1.23 .

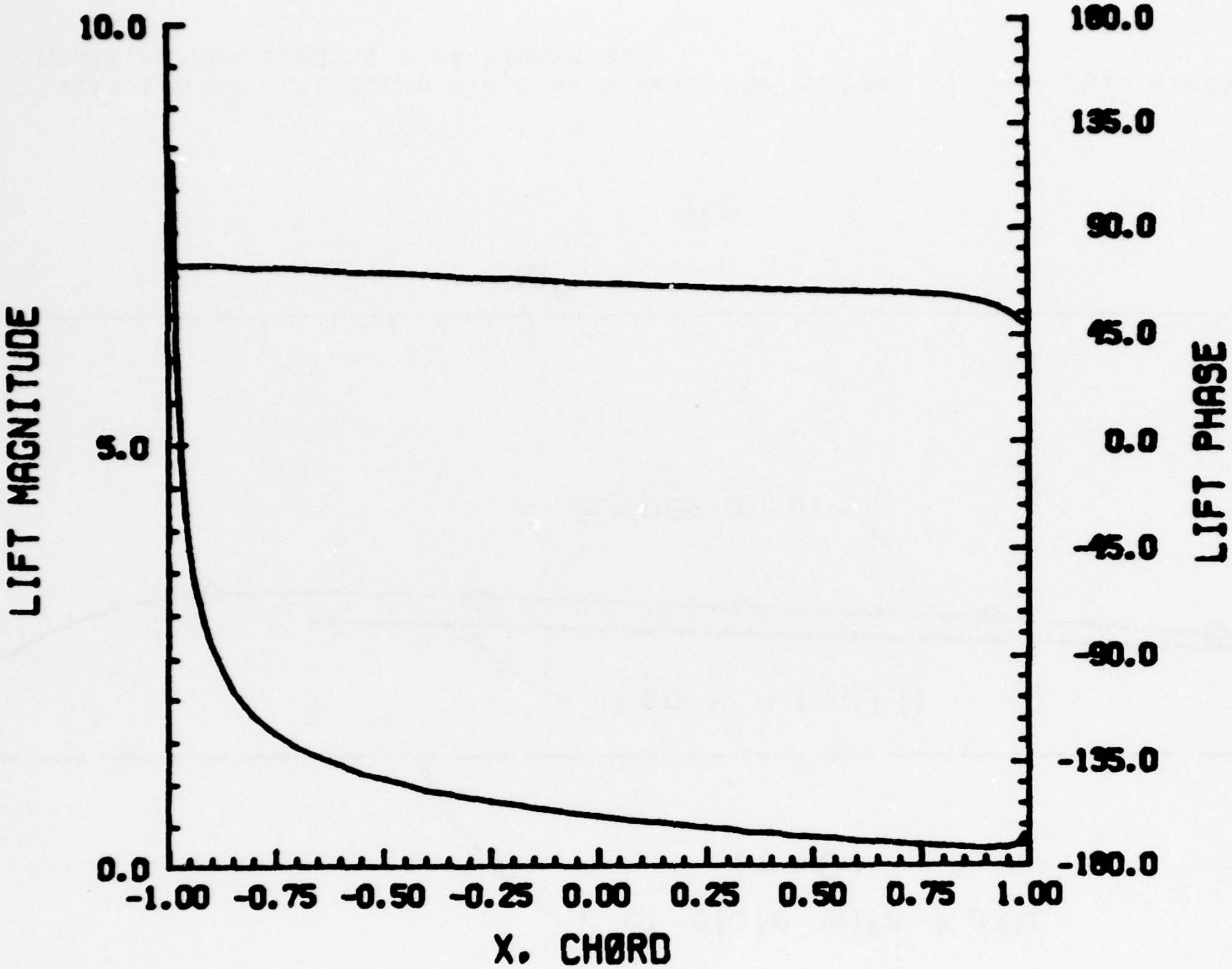

Figure 7. Lift distribution on an airfoil in a sinusoidal gust; Re = 1000, k = 2.0

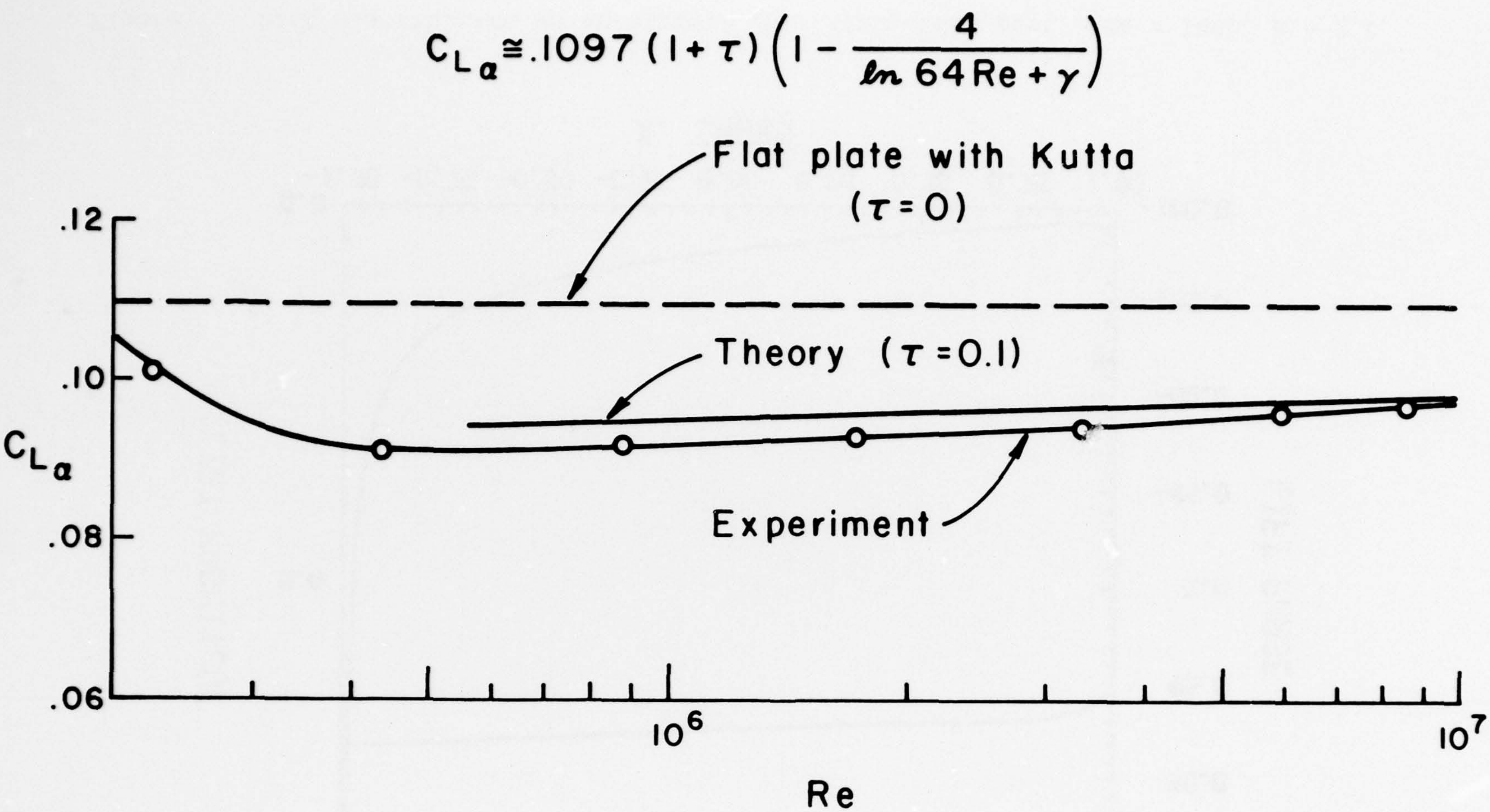

Figure 8. Variation of lift curve slope with Reynolds number; viscous thin airfoil theory modified for leading edge bluntness.

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Dr Yates, in this publication, proposes an original approach to the problem which permits a first calculation of boundary layer effects. He derives a new kernel of the lifting surface integral equation which takes into account shear flow effects in the boundary layer. A very lively discussion took place after the presentation which was appreciated as an important step in the preparation of new Panel activities.

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